#### SM223 · Calculus III with Optimization

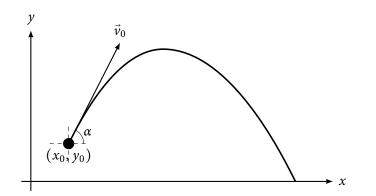
# Lesson 10. Projectile Motion

## 1 In this lesson...

• Describing the trajectory of a projectile with parametric equations

#### 2 Trajectory of a projectile

- A projectile with mass *m* is fired
  - initial point  $(x_0, y_0)$
  - $\circ$  angle of elevation  $\alpha$
  - $\circ$  initial velocity  $\vec{v}_0$
- Assume:
  - Air resistance is negligible
  - The only external force is due to gravity



- Let's derive parametric equations that describe the trajectory of this projectile
- 1. Let's define  $v_0 = |\vec{v}_0|$  (we're just renaming the initial speed, or the magnitude of the initial velocity). Using this new notation, write  $\vec{v}_0$  in terms of  $v_0$  and  $\alpha$ . *Hint*. We'll need to use trigonometry.
- 2. We need an expression for the acceleration  $\vec{a}(t)$  of the projectile.

Recall Newton's second law of motion: if at any time *t*, a force F(t) acts on an object of mass *m* producing an acceleration  $\vec{a}(t)$ , then  $\vec{F}(t) = m\vec{a}(t)$ .

Since the only external force is due to gravity, which acts downward, we have that  $\vec{F}(t) = m\vec{a}(t) = \langle 0, -mg \rangle$ . Solve for  $\vec{a}(t)$ . 3. Using our answer from part 2, write an expression for the velocity  $\vec{v}(t)$  of the projectile.

*Hint 1.* Recall that  $\vec{a}(t) = \vec{v}'(t)$ . *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial velocity is  $\vec{v}_0$ , we have  $\vec{v}(0) = \vec{v}_0$ . Use the expression for  $\vec{v}_0$  we obtained in part 1.

4. Now, using our answer from part 3, write an expression for the position  $\vec{r}(t)$  of the projectile.

*Hint 1.* Recall that  $\vec{v}(t) = \vec{r}'(t)$ . *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial point is  $(x_0, y_0)$ , we have  $\vec{r}(0) = \langle x_0, y_0 \rangle$ .

5. Expand the vector equation we obtained in part 4 to write parametric equations (i.e. x = ..., y = ...) for the trajectory of the projectile.

### 3 Problems

In each of these problems, ignore the possibility of air resistance. Assume that acceleration due to gravity is downward and equal to *g*.

**Problem 1.** A cannon sitting atop of a 200 m cliff shoots a projectile at a speed of 50 m/s and at an angle of 30° above the horizontal. A building 50 m tall sits 300 m from the base of the cliff. Does the projectile strike the building? (Ignore the width of the building).

**Problem 2.** A lacrosse player 80 m from an open goal throws a ball at an angle of 25° above the horizontal with a speed of 20 m/s. Does the ball enter the goal in the air? Assume that the ball leaves the stick 3 m above the ground and that a lacrosse goal is 2 m high.

**Problem 3.** An F-18 is flying at 200 m/s at an altitude of 1500 m and at an angle of 5° below the horizontal when it drops a bomb. There is a 300 m building 3000 m from the point below the F-18 when it drops its bomb. Does the bomb hit the building? (Ignore the width of the building.)

**Problem 4.** In Fenway Park, the Green Monster is a wall approximately 11.3 m tall and 94 m from home plate along third base line. A ball was hit at an angle of 30° along the third base line and barely cleared the Green Monster. At what speed did the ball leave the bat? Assume that batter hit the ball 1 m above the ground.